The Natural Period of Linear Conductors

By C. R. ENGLUND

SYNOPSIS: This paper describes the experimental determination of the frequency of free electrical oscillation of straight rods and circular loops. The results agree more closely with the formula of Abraham than with that of MacDonald. For three rods whose lengths were 300 cm., 250 cm. and 227.1 cm., the ratio of wave length at resonance to rod length had the values 2.11, 2.13 and 2.13, respectively. Measurements taken upon 250 cm. rods bent into circular arcs of different radii gave values of the ratio of resonant wave length to arc length which passed through a minimum value and were virtually independent of the radius of the arc over a wide range, deviating markedly only at the extreme value of minimum radius possible and infinite radius. The extreme measured range of the ratio was 2.05 to 2.166. The wave lengths were measured upon a pair of Lecher wires and a very satisfactory meter for the rapid comparison of waves of short length was found to be a quarter wave length Lecher frame. This frame showed a constant end correction so that $\lambda = 4(d + 3.1)$, d being the length of the parallel rods.

IN 1898 Abraham [1] calculated the free period of an extended but relatively narrow metallic ellipsoid of revolution when excited by an electrical impulse. To a good approximation the fundamental natural period found was related to the major axis length by the expression $\lambda/l = 2$. Obviously a rectilinear conductor of circular cross-section cannot differ markedly from such an ellipsoid and Abraham concluded that the equation $\lambda/l = 2$ was also valid for this.

In 1902 Macdonald [2] arrived, by a theoretical deduction quite different from that of Abraham, at the expression $\lambda/l = 2.53$ for the fundamental free period of a linear conductor. Moreover Macdonald assigned the same value to the linear conductor when bent into a nearly closed circle. In the next twelve years a variety of papers were published [3] giving results which were aimed at clearing up this discrepancy without however definitely settling the matter one way or the other. The subject has in recent years become of interest again following the development of the short wave vacuum tube oscillator and the conjoint use of rectilinear conductors as radiators (or "reflectors")—particularly in grids of parabolic form.

Since the universal method of measuring wave length is that of determining the nodal distances for standing waves on parallel con-

[1] Abraham, *Ann. der Phy.*, 66, 435, 1898.

[2] Macdonald, "Electric Waves," pp. 111–112.

[3] See Bibliography at end.

ductor "Lecher" systems, and since further there are practical advantages in reducing the total length of these systems to a single or half a single nodal length ($\lambda/2$ and $\lambda/4$ "Lecher" frames), the problem of the natural period of a rectilinear conductor can be broadened to include a study of the shortest favorable shape of a linear conductor for use as a wave length standard. It is the purpose of this paper to give the results of some experimental work relating to both these questions. At the same time an examination of the operation of an

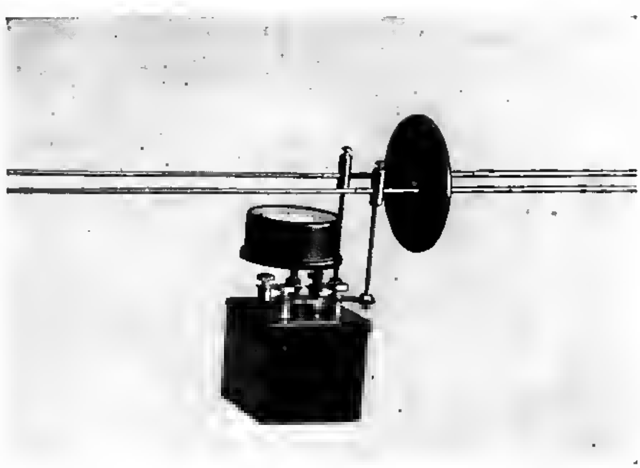


Fig. 1

extended Lecher system as a basic wave measuring apparatus was a necessary preliminary.

It was a matter of only a small amount of experimentation to demonstrate that the practical Lecher system for wave length measurement would necessarily consist of a pair of heavy uniformly spaced copper wires devoid of insulator spacers and at least a couple of wave lengths long. While the attenuation of Lecher systems made of ordinary wire is not great, as attenuations go, the accuracy with which the nodal points can be located, in the manner later described, depends markedly on the degree in which space resonance currents build up, and it is quite necessary to supply sufficient copper. The wire used here was No. 8 B. & S. gauge (3.26 mm. diam.) soft drawn copper and by "ironing" it with a slotted wood piece it was made as smooth as was necessary. It was stretched between two poles out of doors and kept tight with a turnbuckle. The spacing was fixed at 5.15 cms. by a metal bridge at one end and a micarta bridge at the other. The

total length was 15.4 meters. A photo of the sliding bridge unit is given in Fig. 1.

The method of using such a Lecher system requires consideration. If we set up a pair of parallel wires and feed energy from a generator into them, we shall have, unless the far end of the wires is terminated by the "surge" impedance of the line, a standing wave system set up. This standing wave will be most pronounced when the outer end is so terminated as to return all the energy arriving there and this requires that the terminating impedance be a pure reactance. If the extreme values of zero or infinite reactance be chosen, a current anti-node or node will respectively occur at the far end.

If the far end be reactively terminated and we observe the current distribution while moving back towards the generator, we shall find the standing wave persisting up to the generator itself. However, if the line be dissipative, the returned wave will not completely cancel the outgoing wave at phase equality locations and the current maxima and minima will become less contrasty. If the line be practically non-dissipative, the maxima and minima will not deteriorate as we approach the generator.

The returning wave is re-reflected at the generator end and traveling to the far end returns once more, this process being repeated until its amplitude has faded out. Usually the generator appears as a resistive impedance when viewed from the line so that not much energy survives reflection at this end. In any case, as the generator is the primary energy source, the generator voltage introduced into the line and the voltage of the re-reflected waves add vectorially to give a component just sufficient to maintain the standing wave line current. By an adjustment of the effective line length, either by changing the physical length, the far end reactance, or the generator impedance as viewed from the line, the power input to the line may be maximized for the particular generator used.

When this state of affairs has been attained, the standing wave may be observed by either a current or voltage operated device moved along the line. (If this device absorbs too much energy, it becomes a source of disturbing reflections itself, complicating matters by superposing on the original standing wave another pair of standing waves. It is not advisable to permit such secondary waves to exist in measurable amplitude.) Necessarily the standing wave is closely sinusoidal and at the maximum values $\partial I/\partial l = 0$ so that locating these current extremes is not an accurate experimental process. The

accuracy of determination of the distance between two consecutive values of $\partial I/\partial l = 0$ will not be sufficient unless very great care be taken, a large line current supplied, and an indicator responding well to $\partial I/\partial l$ (such as a square law thermocouple) be used. For the attainment of greater accuracy an average over a number of nodal distances must be used. In short, this method of measurement requires a relatively long line for accuracy, up to the limit where a deterioration of the maxima and minima has become pronounced due to attenuation. At the current minima $\partial I/\partial l$ is great enough for good settings but no meters of requisite sensitivity at the zero end of their scales exist.

Another and more sensitive method of observation of nodal distances is to make use of the variation of line current as the total line length is varied, particularly if both ends of the line be pure reactances and the line conductors have an adequate copper content and very good insulation. Coupling such a line weakly to a generator by merely placing the generator in the neighborhood makes it possible to build up very sharply resonant standing waves so that settings without any particular precautions can be made to one part in 3,500. Of course the point located is again one where $\partial I/\partial l = 0$ but the value of ΔI, for a given value of Δl, is very much larger. Moreover the accuracy is not decreased by shortening the line [4] and, since the resonance energy is then dissipated in a shorter length of line, the corresponding increase in the resonance current makes the antinode easier to locate. Although this method is very sensitive to energy losses it is by far the best method of using a Lecher system. Essentially it is nothing but a sharp "tune" observed and interpreted as space resonance. In the present work the length of the Lecher system was sufficient to observe four resonance maxima throughout the range of wave lengths used, giving, by difference, three wave length readings. These readings could readily be duplicated to one part in 3,500; it is improbable however that the velocity of propagation along the wires is within one part in 3,500 of that of free space so that this precision is unusable, though comforting. This accuracy of setting would be useful where small frequency differences or line constant changes are to be observed. Since the measurements checked admirably from day to day and the line attenuation was low, it was assumed that the line velocity was not affected by the adjacent ground (110 cms. at lowest point) and was near enough to that of the velocity of light to allow the line to be used as the basic wave length standard.

[4] A. Hund, Sci. Paper No. 491, Bureau of Standards, 1924, points out the same fact.

Resonance Period of a Straight Rod

It was at first thought that it would be relatively simple to set up a vacuum tube generator together with a rectilinear rod and run resonance curves on the latter. This did not prove to be the case however. Working indoors was impossible and all the apparatus had to be moved out of doors. When the two antennas (the generator

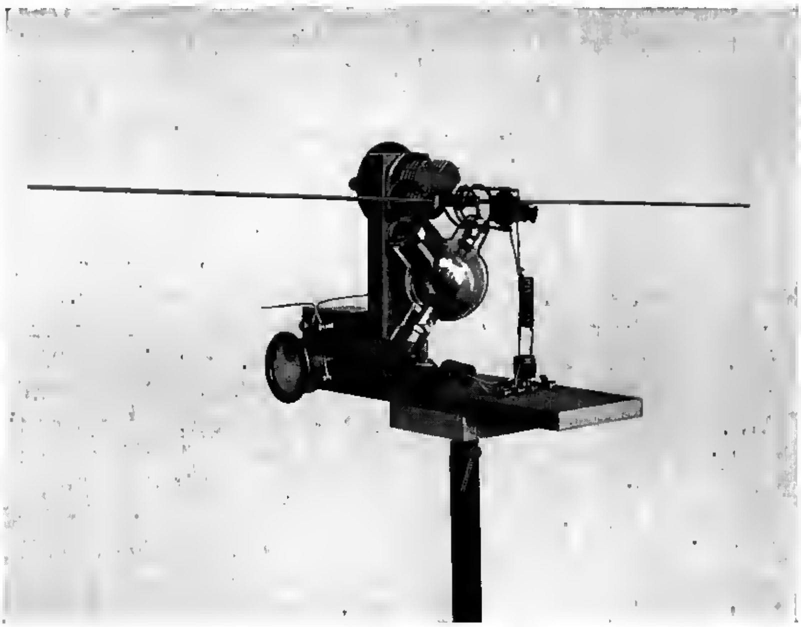


Fig. 2

antenna and resonant rod) were mounted vertically, the operator became a mobile reflector himself seriously disturbing the transmission. Moreover the ground was unsymmetrically disposed with respect to the two antenna ends and this was felt to be a disadvantage. With the antennas horizontal these objections vanished but the reflection from the ground had to be taken into account. This latter was the arrangement finally adopted and is shown in Fig. 3, the apparatus in question being mounted on the two tripods.

The attempt was made to get a generator whose radiation field was constant over a wide wave length range so that the rod resonance wave length could be obtained by observation while turning the generator tuning condenser. As a matter of fact, the generator output was apparently satisfactorily constant while actually not so,

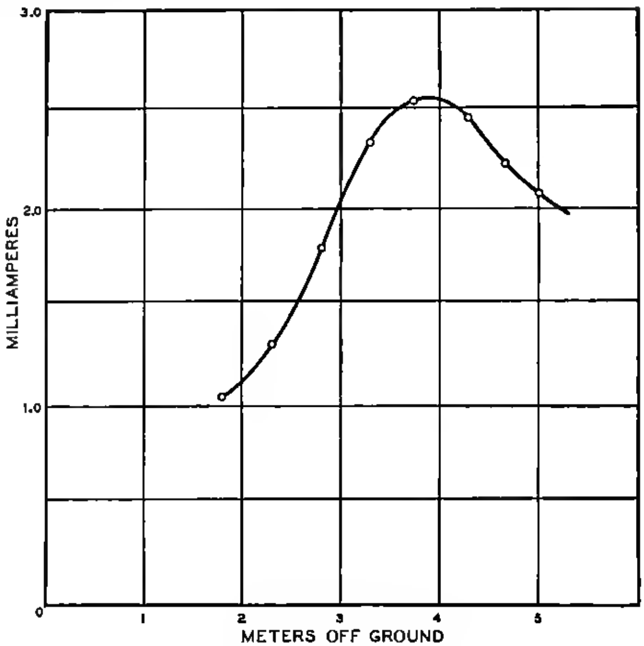
and some time was wasted trying to get consistent results. Finally a control meter was placed on the generator and resonance curves run, observing by small steps wave length, rod meter deflection, and control meter deflection. Then by reducing the rod meter deflection to a standard control meter value satisfactory observations were obtained. As it turned out, the averaged value of all the unsatisfactory observations checked the resonance curve value very closely, but the individual observations scattered all over the rather broad resonance curve top. To avoid the reaction of antennas upon each other they must be separated by at least a wave length, and such a

Fig. 3

separation here resulted in too low field strengths unless the antennas were off the ground sufficiently to get an additive combination of the direct and earth reflected radiations. The resonant rod should in any case be well off ground to make certain that its period is not affected by the ground. Check tests showed that at 4 meters distance the ground did not affect the free period markedly. It would be much simpler to observe the resonance curve of a variable length rod, at constant wave length, but it would not be permissible to use a rod of variable diameter and a telescoping arrangement is the only practical method of obtaining collapsibility.

The generator used was an UX852 tube connected as in Fig. 2. By means of the variable condenser shown a wave length range of 4.24 to 8.44 meters was obtained. This condenser is a cut down "Remmler," the oscillating coil is a three turn center-tapped unit

of 1/8 inch (0.32 cm.) copper tubing, the choke coils are 10 microhenry units resonant at 6.1 meters and the antenna rods are connected directly across the resonant circuit. This apparatus was finally set up on a tripod raising the antenna 2.55 meters above ground. The control meter consisted of a pair of 15 cm. wires connected to a thermocouple and Weston model 301 micro-ammeter combination. It was not resonant in the generator range and was fastened on the generator base permanently.



Curve I—300 cm. rod

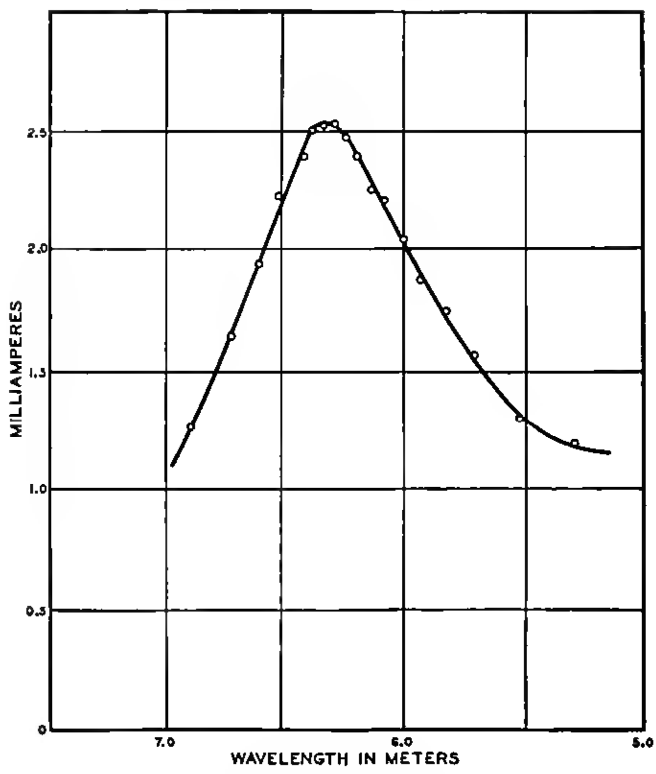
The rods studied were ordinary 1/2 inch copper and brass tubes and were straightened and mounted in a bracket consisting of a pair of phosphor bronze knife edges spaced 27.5 cms. apart. These knife edges were attached to a micarta strip and connected to a 600 ohm thermocouple and Weston model 301 micro-ammeter combination as for the control meter. A tripod supported the rod mount and the meter was read with a field glass at the generator site. The wave length was determined by a $\lambda/4$ Lecher "frame" (see Fig. 5) calibrated by means of the Lecher wires. This will be described later.

A full scale deflection of the resonant rod micro-ammeter corresponded to 4 milliamperes heater current and this was shown, by

means of experiment with two rods each equipped with a removable thermocouple, to have no effect on the resonant frequency. With two rods in proximity the resonance was much more sharp and definite than for one rod, and was also more sensitive to effects producing variations in the natural period.

Three separate rods were used, their specifications being,

Length	O.D.	I.D.	Material
300 cm.	1.27 cm.	1.02 cm.	Copper
250 "	1.27	1.02	"
227.1 "	1.27	1.02	Brass



Curve II—300 cm. rod

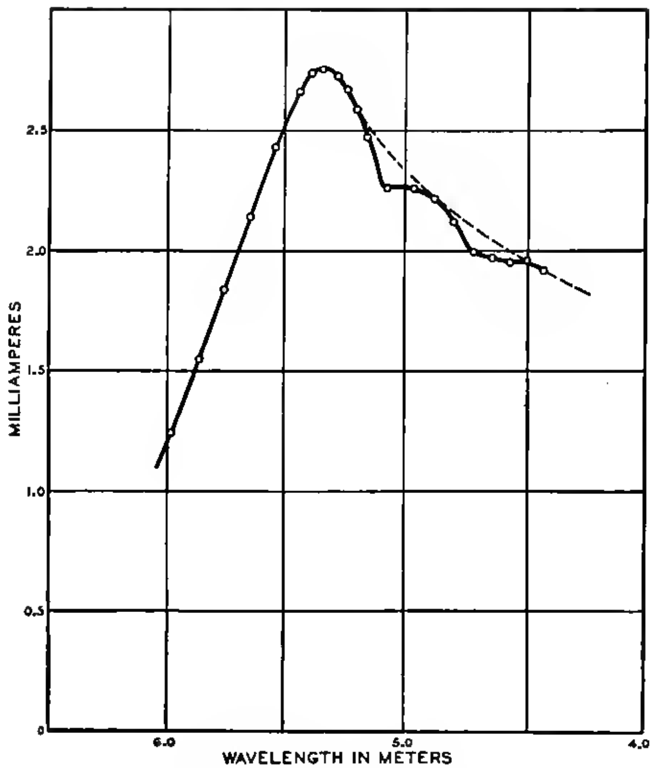
Curve I shows the maximum current versus height above ground relation for the 300 cm. rod. The generator antenna was 2.55 meters above ground and the horizontal spacing between generator antenna and resonant rod always 7.7 meters, both antennas horizontal. The

curve thus needs correction for the fact that the air line distance varied as the rod was lowered and raised. This correction is small however.

It will be observed that the maximum current occurs at 3.85 meters and taking this value for determining the distance to the reflecting ground surface gives, if we assume a radiation field only,

$$(1.3 + 2h)^2 + (770)^2 = \left(\sqrt{130^2 + 770^2} + \frac{\lambda}{2}\right)^2$$

or with $\lambda = 6.34$ meters, $h = 3.26$ meters. The difference $3.26 - 2.55 = 0.71$ meter is the apparent distance under ground of a metallic sheet equivalent to the ground itself.



Curve III—250 cm. rod

Curves II and III give the resonance curves of the 300 and 250 cm. rods respectively, mounted horizontally and 4 meters above ground. The latter curve is complicated by two extraneous resonances

occurring somewhere in the generator circuit. With the advent of inclement weather the source of these resonances could not be looked for. These dips in the resonance curve are not evident to an observer watching the resonance rod meter as the generator wave length is varied; they become evident only on plotting a carefully taken resonance curve. The results both for preliminary eye settings and final resonance curve are:

Rod	By Res. Curve		By Eye Settings	
	λ	λ/l	λ	λ/l
300	6.34	2.11	6.33	2.11 (av. 10 settings)
250	5.36	2.14	5.32	2.13 (av. 13 ")
227.1	—	—	4.84	2.13 (av. 8 ")

A check eye setting on the 250 cm. rod mounted vertically agreed with the other eye settings. The eye settings, or eye estimates of the top of the rod resonance curve, were made first, the resonance curves were run last. On discovering the "dips" in the 250 cm. rod curve it became useless to run a 227.1 resonance curve before eliminating these dips, the preliminary curve being discarded. It is not certain however that these "dips" were present in most of the eye settings as these were chiefly made with the generator nearer ground and with shorter power leads. They are given for what they are worth.

Evidently experiment more nearly checks Abraham [5] than Macdonald, the rods operating as if their effective lengths were 6–7 per cent greater than their physical lengths. Whether this "end correction" varies with the rod diameter was not investigated owing to bad weather. Several rods were however bent into circles, cut to 250 cms. perimeter (outside length), and their natural periods determined. The results follow below. With the bending, the radiation resistance of the rods went down pronouncedly and the knife blade contacts were shortened to span a distance of 10.2 cms. A preliminary test showed the natural period of such rings to be independent of their orientations; they were therefore hung up in the knife edges with open gap downwards. The top edge of the ring was always

[5] Abraham gives a correction term of the form

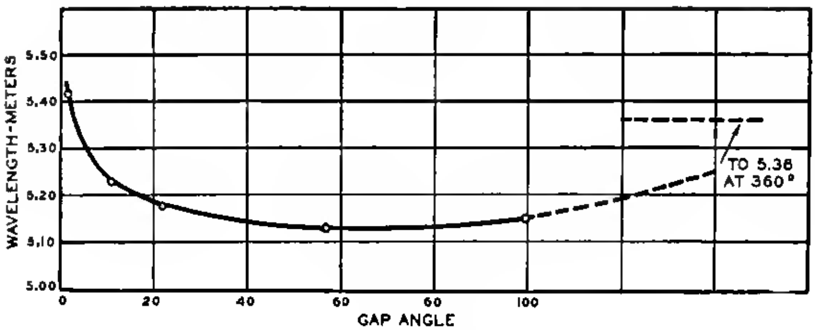
$$\frac{\lambda}{l} = \frac{2}{n}\left[1 + C_n \cdot \left(\frac{1}{4 \log \frac{2l}{d}}\right)^2\right],$$

where "n" is the order of the harmonic and C_n a complicated integral. For "n" = 1 and the 300 cm. rod, $\lambda/l = 2.018$, which is too small.

3.17 meters above ground, generator antenna horizontal and 2.55 meters above ground, horizontal separation between ring plane and transmitting antenna 7.2 meters. The results were:

Ring Shaped 250 cm. Rod with Gap

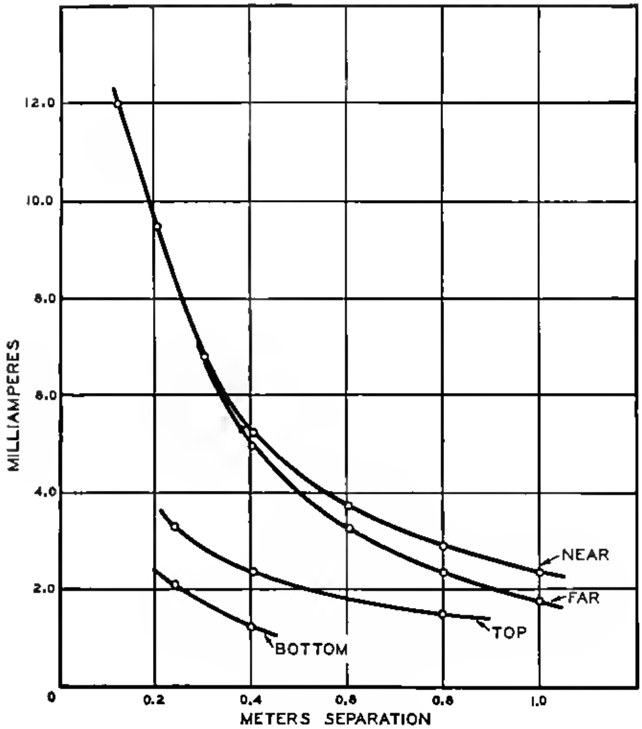
Gap Angle	Ring Radius	λ Meters	λ/*l*
1.58°	40. cms.	5.414	2.166
10.8	41.1	5.228	2.09
21.8	42.4	5.178	2.07
56.4	47.2	5.128	2.05
99.8	55.1	5.148	2.06
360.0	∞	5.36	2.14



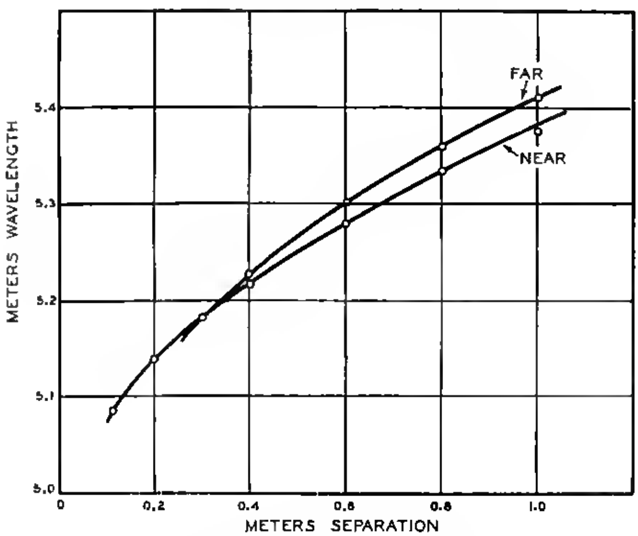
Curve IV

These values are plotted in Curve IV and do not check Macdonald's conclusion earlier referred to. In fact the first part of the curve is most simply explained by viewing the rod as a resonant inductance whose tuning capacity is decreased as the rod gap opens. The existence of a minimum resonant wave length is less easy to explain in this manner.

It was early observed that the current amplitude and sharpness of resonance of a rectilinear rod were greatly increased by the proximity of a second rod. Curves were therefore run with two parallel rods arranged both in horizontal and vertical planes, the spacing being changed while current magnitude and resonance wave length were observed. In each case the rods were symmetrically mounted about a central point held at the height of the generator antenna above ground (2.55 meters) and the short knife edge support, mentioned in connection with the rings, was used. All the antennas were horizontal and as accurately parallel as it was possible to set them. Fig. 4 shows one mounting and Curve V the current versus spacing relation, while Curve VI gives the resonance wave length versus spacing.



Curve V—Two rods each 250 cm. long



Curve VI—Two rods each 250 cm. long

It is unwise to attempt any critical conclusions from these curves as long as the standing wave pattern of the direct and earth reflected wave interference is not known. But two facts seem clear, viz.: a closely spaced rod pair gives a marked current step up over that of a single rod, and the natural period of a rod pair approaches the value $\lambda = l/2$ as the spacing decreases. It is obvious that the currents

Fig. 4

in the two rods, at close spacing, are nearly anti-phased and that their vector sum must be nearly that current which an isolated rod would carry. The analogy with an anti-resonant circuit is evident, the "stepped up" current being limited only by the ohmic losses in the rods. Actually the tune, at close spacings, was excessively sharp and hard to set for with the generator condenser. The exigencies of the mounting of the rods and the observing of the meter prevented observations at close spacing for the "far," "top" and "bottom" meter positions.

High Frequency Wave Meter

A pair of Lecher wires constitutes a wave measuring system much too awkward and extended for rapid use, and some apparatus much more portable and speedy in operation is necessary; especially when running resonance curves. Such an apparatus is a pair of heavy uniform and parallel conductors arranged with one or two sliding

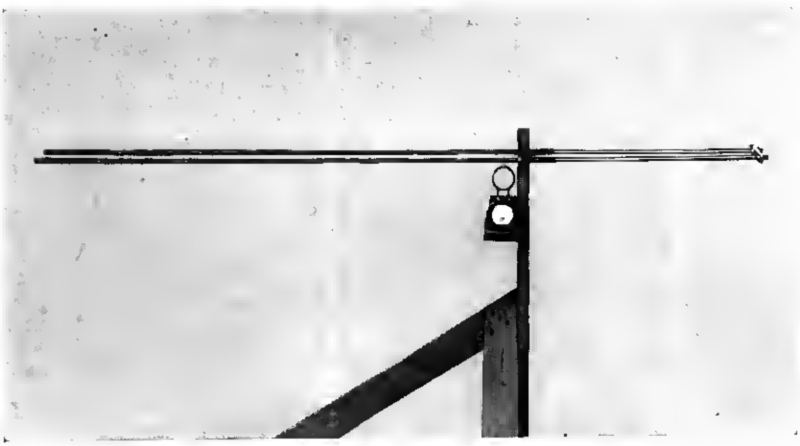


Fig. 5

metallic short-circuiting discs so as to constitute a quarter or a half wave length "Lecher frame" (see Fig. 5). Such a frame, if containing sufficient copper, is very sharply resonant, need only be a quarter wave length long, and after calibration becomes a wave length meter indicating to a precision of 1 part in 2,500 with the greatest ease. As an accessory apparatus such a wave frame was constructed, calibrated, tested for factors affecting its accuracy and used for most of the measurements reported above.

The Lecher frame shown in Fig. 5 was made of a pair of straight copper tubes 1.27 cms. diameter spaced 10.1 cms. center to center and sliding through a brass disc 15.5 cms. diameter, 0.3 cm. thick, with inserted guide tubes. It had a workable wave length range of 4 to 7.5 meters and the resonance setting was indicated by a three turn coil-thermocouple-microammeter combination placed with coil clearing one of the rods at the disc end by approximately two cms. It was ordinarily cleaned to make good contact at the disc guides but at no time was any indication noticed of an apparent lengthening of the frame due to a moving back inside of the guides of the contact point. It was calibrated over its whole range in terms of the Lecher

wire system already mentioned, calibrated not once but various times and unfailingly indicated 3.1 cms. too short. That is, the distance "d" from open end to disc was 3.1 cms. short of $\lambda/4$, or $\lambda = 4(d + 3.1)$. It was not found possible to make a trombone slide which maintained its spacing accurately, and before each reading was completed a paper template was laid on the open end and the tubes given a slight bend to obtain parallelism. The setting was then completed. This process was much less bothersome than might appear from its description.

A 35 x 44 cm. copper plate was clamped to the brass disc to increase its effective area. This brought the 3.1 cm. correction down to 2.67 cms. (measured at 5.29 meters wave length). The end effect is therefore chiefly due to the open end. No doubt it will change if the spacing of the Lecher frame is changed; the fortunate feature is that it appears constant for a given spacing.

To obtain an idea of the effect of a slight degree of non-parallelism and the presence of insulation material, the rods were first bent out, then in, next a 1.3 x 3.8 x 12.7 cm. hard rubber block was laid flat across the outer end and finally this block was laid across the middle of the frame. The results were:

Lecher Frame Length	Condition	True $\lambda/4$ (Av. of Four Settings Each)	Deficiency in Frame
122 cms.	Parallel	125.07 cms.	3.07 cms.
"	Diverge 0.2 cm.	125.04	3.04
"	Converge 0.2 cm.	125.20	3.20
"	Rubber plate at end	126.21	4.21
"	Rubber plate at middle	125.58	3.58

Obviously a slight divergence is an advantage rather than otherwise, for an uncalibrated frame, and dielectric spacers at high potential points are sources of noticeable error.

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